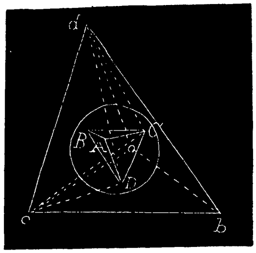
II. Solution by B. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, O.

Let $ABCD$ be the tetrahedron about which is circumscribed the sphere ; and $abcd$ the tetrahedron formed by planes tangent respectively at A, B, C, and D. Given, also, that $BC.AD=CA.BD=AB.CD$.



To prove that Aa, Bb, Cc, Dd meet in a point, as at O.

The points c, D, C, and d are in the same plane. For D and C have their respective distances from the planes cbd and cad in the same ratio, viz : AD^2/AC^2 and BD^2/BC^2, which are equal, since by hypothesis $BC.AD=AC.BD$.

That first part of preceding statement is true is evident from the fact that if diameters are supposed to be drawn from A and B respectively, the respective projections upon these diameters of the chords AD, AC, and BD, BC, have the same ratios respectively as the squares of the chords ; and the projections equal the distances respectively from the points D and C to the planes.

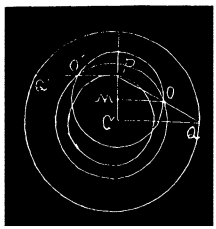
Similarly, co-plainar are the points c, B, C, b ; b, A, B, a ; and a, D, A, d.

Therefore, Dd and Cc intersect ; so, also, Dd and Bb, and Cc and Bb.

113. Proposed by T. W. PALMER, Professor of Mathematics, University of Alabama.

Given three concentric circles. Draw a straight line from the inner to the outer circumference that shall be bisected by the middle circumference.

I. Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, O., and J. C. GREGG, A. M., Superintendent of Schools, Brazil, Ind.



Let C be the common center and CP the radius of the inner circle, and CQ that of the outer circle.

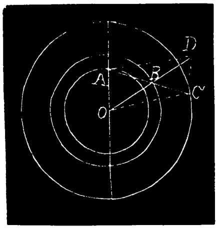
Bisect CP at M and with one-half of CQ as radius and M as center describe a circle cutting the middle circumference at O or O'. Draw PO and produce to the outer circumference at Q. Then POQ is the required line. For, $\because PM=MC$ and $MO=\frac{1}{2}CQ$, $\therefore PO=OQ$.

According as $\frac{1}{2}$radius of outer circle is greater than, equal to, or less than the radius of the middle circle increased by $\frac{1}{2}$radius of inner circle, there are two solutions, one solution, or no solution.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.; J. SCHEFFER, A. M., Hagerstown, Md.; COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.; J. C. NAGLE, C. E., Professor of Civil Engineering, Agricultural and Mechanical College of Texas, College Station, Tex.; CHAS. C. CROSS, Libertytown, Md.; J. O. MAHONEY, B. E., M. Sc., Teacher of Mathematics and Science in Carthage High School, Carthage, Tex., and the PROPOSER.

Let O be the center of the circles, and A any point on the inner circumference. With O as a center and a radius$=2$ times the radius of the middle circle, and with A as a center and a radius=the radius of the outer circle, describe two arcs intersecting at D. Draw OD intersecting middle circumference at B.

Through A and B draw AC intersecting outer circumference at C. Then $AB=BC$, and AC is the required line.



PROOF. $OB=BD$, $OC=AD$, and $\angle OBC=\angle ABD$. $\therefore \triangle OBC=\triangle ABD$. $\therefore AB=BC$. Also, $OADC$ is a parallelogram, of which the diagonals OD and AC bisect each other at B.

COROLLARY 1. Put a, b, and c=the respective radii of the three concentric circles, taking $a<b<c$, and put $2d$=line AC. Then, from the relation of the diagonals to the sides of a parallelogram,

$$2d=\sqrt{(2c^2+2a^2-4b^2)}.$$ GRUBER.

COROLLARY 2. The problem is possible only for $c-b=b-a$ to $c-b=b+a$, or for $2b=c+a$ to $2b=c-a$. Whence the limits of $2d$ are $c-a$ and $c+a$, the parallelogram in either case reducing to a straight line. GRUBER.

COROLLARY 3. The point D is without the outer circle for $2b>c$. When $2b=c$, D lies in the outer circumference. When $2b<c$, D lies within the outer circle. GRUBER.

COROLLARY 4. When $2b>\sqrt{(c^2+3a^2)}$, $2d$ lies wholly without the inner circle. When $2b=\sqrt{(c^2+3a^2}$, $2d$ is tangent to the inner circumference. When $2b<\sqrt{(c^2+3a^2)}$, $2d$ is a secant of the inner circle. GRUBER.

Also solved by *G. B. M. ZERR, J. W. YOUNG. GEORGE R. DEAN, B. F. YANNEY, B. F. SINE, WALTER H. DRANE*; and *WM. K. NORTON*.

CALCULUS.

88. Proposed by JOHN M. ARNOLD, Crompton, R. I.

When a watch is wound up, the mainspring is closely coiled around a cylindrical piece called the hub of the barrel-arbor. When entirely run down the spring forms an annulus against the inner circumference of the barrel. Show that if the width of the annulus is a little more than one-fourth of the radius of the barrel, the spring will run the watch the greatest number of hours at one winding, the diameter of the hub being one-third the inside diameter of the barrel.

I. Solution by the PROPOSER.

Let R=radius of the barrel, r=radius of the hub, t=thickness of spring, x=width of the annulus when run down, y=width of the annulus when wound up, u=number of turns required to wind the spring.

Then x/t=number of coils of the spring when run down, and y/t=number of coils when wound up.

Hence $u=y/t-x/t$......(1).

It is evident that the area on the bottom of the barrel covered by the spring will be the same in the wound or unwound condition.

Hence $R^2\pi-(R-x)^2\pi-(r+y)^2\pi-r^2\pi$.

Reducing $2Rx-x^2=2ry+y^2$..... (2).

From (1) and (2), $tu=\pm\sqrt{[2Rx-x^2+r^2]}-r-x$.